Mechanisms, Structural Closure, and Proto-Closure in the UNNS Substrate

UNNS Research Collective

Definition 1 (Numerical Object)**.** *A numerical object is an entity whose identity is fully determined by its value in a numerical field, independent of the generative process by which it is obtained.*

Definition 2 (Collapse)**.** *Collapse is a transformation acting on objects that preserves invariant structure while eliminating non-invariant features under refinement.*

Definition 3 (Mechanism)**.** *A mechanism is a tuple*

$$M = (\Sigma, R, C, \mathcal{O}),$$

where:

- Σ *is a finite symbolic alphabet,*
- R *is a set of generation or rewrite rules on* Σ*,*
- C *is a set of admissibility constraints,*
- $\mathcal{O}$ *specifies the ordering or interaction of rule applications.*

Definition 4 (Structural State)**.** *A structural state is any equivalence class of symbolic configurations generated by a mechanism, where equivalence is defined by admissible renaming and canonicalization rules.*

Definition 5 (Refinement)**.** *A refinement is an operation that increases structural resolution by one or more of the following:*

- *increasing generation depth,*
- *tightening admissibility constraints,*
- *reducing equivalence tolerances,*
- *increasing rule application strictness.*

Definition 6 (Refinement Sequence)**.** *Given a mechanism* M*, a refinement sequence is a sequence of structural states*

$$S_0 \to S_1 \to S_2 \to \cdots$$

generated by successive refinement operations applied to M*.*

Definition 7 (Structural Closure)**.** *A mechanism exhibits structural closure if its refinement sequence enters a nontrivial recurrent orbit in the space of structural states, up to structural equivalence.*

Definition 8 (Trivial Termination)**.** *A refinement sequence terminates trivially if it halts due to exhaustion, collapses to a null structure, or freezes into a fixed configuration without recurrence.*

Definition 9 (Proto-Closure)**.** *A mechanism exhibits proto-closure if it exhibits structural closure prior to any numerical evaluation of its generated structures.*

Definition 10 (Structural Collapse)**.** *A mechanism undergoes structural collapse if, under refinement, it fails to exhibit structural closure.*

Proposition 1. *Numerical objects are not preserved under collapse.*

Proof. Collapse eliminates numerical identity while preserving only invariant structural features. Since numerical objects are defined solely by value, they are not invariant under collapse. □

Proposition 2. *Any object preserved under collapse must admit a representation as a mechanism.*

Proof. Collapse acts on generative structure rather than on evaluated outcomes. Any preserved object must therefore retain a pre-evaluative generative description, which is precisely a mechanism. □

Proposition 3. *Structural closure excludes trivial termination.*

Proof. Trivial termination yields no recurrent orbit in structural state space. Since recurrence is required by definition, trivial termination does not constitute closure. □

Proposition 4. *Proto-closure is a necessary condition for a mechanism to be preserved under collapse.*

Proof. Collapse removes numerical identity. Preservation must therefore occur at the pre-numerical level. A mechanism that does not exhibit proto-closure cannot survive collapse under refinement. □

Proposition 5. *Structural collapse may occur independently of numerical instability.*

Proof. Structural collapse depends on failure of recurrence under refinement. Numerical instability is neither required nor sufficient for such failure. □

Remark 1. *Proto-closure does not imply numerical convergence, physical relevance, or observational correspondence.*

Remark 2. *Distinct numerical objects may correspond to the same mechanism and are therefore indistinguishable under collapse.*

Remark 3. *Structural closure is invariant under admissible renaming of symbols and reparameterization of generation rules.*

Remark 4. *Proto-closure is a structural property of mechanisms, not an ontological claim about physical reality.*

Remark 5. *The framework presented here constrains admissible investigations of fundamentality to mechanism-level analysis.*

Appendix A: Structural Equivalence, Refinement Metrics, and Collapse Projection

Definition 11 (Structural Equivalence Metric)**.** *A structural equivalence metric is a function*

$$d : \mathcal{S} \times \mathcal{S} \to \mathbb{R}_{\geq 0}$$

on the space of structural states $\mathcal{S}$ satisfying:

- $d(S, S) = 0$,
- $d(S_1, S_2) = d(S_2, S_1)$,
- $d(S_1, S_3) \leq d(S_1, S_2) + d(S_2, S_3)$,

and invariant under admissible symbol renaming and canonicalization.

Definition 12 (Refined Structural Closure)**.** *A mechanism exhibits structural closure if there exist $\varepsilon > 0$ and an integer $k \geq 1$ such that for all sufficiently large n,*

$$d(S_n, S_{n+k}) < \varepsilon,$$

and the recurrent orbit contains more than one equivalence class of structural states.

Remark 6. *This definition excludes trivial termination, fixed-point freezing, and null collapse while admitting periodic and bounded recurrent behavior.*

Proposition 6. *Numerical collapse signatures are projections of underlying structural collapse behavior.*

Proof. Numerical collapse operates on evaluated realizations of mechanisms. Distinct numerical realizations of a single mechanism share the same underlying generative structure and therefore project the same collapse behavior under numerical evaluation. □

Remark 7. *Determining whether a mechanism exhibits proto-closure may be undecidable in general. The framework requires only bounded empirical testability under finite refinement.*

Appendix B: Transient Structural Recurrence and Meta-Instability

This appendix introduces a diagnostic refinement to the structural analysis framework, motivated by empirical behavior observed in Chamber XXIX. The concepts defined here do not modify the core definitions of structural closure, proto-closure, or collapse. They introduce an auxiliary notion intended solely for classification and interpretation.

Definition 13 (Transient Structural Recurrence)**.** *Let M be a mechanism with refinement sequence*

$$S_0 \to S_1 \to \cdots \to S_N.$$

We say that M exhibits transient structural recurrence *if there exist integers $n_1 < n_2 \leq N$, an integer $k \geq 1$, and $\varepsilon > 0$ such that*

$$d(S_m, S_{m-k}) < \varepsilon \quad \text{for all } m \in \{n_1 + k, \ldots, n_2\},$$

and the recurrence condition fails at the terminal refinement level N for all k.

Remark 8. *Transient structural recurrence captures the presence of a non-trivial recurrent orbit over a finite refinement interval that does not persist under continued refinement. It is a property of the refinement history, not of the terminal state.*

Definition 14 (Meta-Instability)**.** *A mechanism is said to exhibit* meta-instability *if it undergoes transient structural recurrence and ultimately undergoes structural collapse.*

Remark 9. *Meta-instability is not a third outcome alongside proto-closure and collapse. It is an observational qualifier that applies only to mechanisms classified as structurally collapsing. In particular, meta-instability does not constitute a form of preservation under collapse.*

Proposition 7. *Meta-instability does not weaken the collapse criterion.*

Proof. By definition, a meta-unstable mechanism fails to satisfy the persistence condition required for structural closure. Therefore, it is classified as structurally collapsing under the criteria of Appendix A. The detection of transient recurrence introduces no additional preservation class and does not alter the verdict of collapse. □

Remark 10. *Empirically, meta-instability distinguishes mechanisms that briefly approximate structural closure from those that never exhibit recurrence. This distinction is diagnostically useful but carries no ontological weight.*

Remark 11. *The existence of meta-instability suggests that apparent stability observed at finite resolution or shallow refinement may be illusory. This provides a structural explanation for phenomena that appear stable at one level of analysis but fail under refinement.*

Appendix X: Determinism, Meta-Instability, and Structural Diagnostics

This appendix records clarifications required for the operational interpretation of structural closure and collapse as implemented in Chamber XXIX. These clarifications do not modify the definitions or propositions of the main text, but ensure that empirical evaluations are reproducible, falsifiable, and conceptually aligned with the underlying theory.

Definition 15 (Deterministic Mechanism Evaluation)**.** *A mechanism evaluation procedure is deterministic if, for a fixed mechanism specification M and fixed evaluation parameters, the refinement sequence $\{S_n\}$ is uniquely determined and reproducible across repeated executions.*

Remark 12. *Determinism is required for structural evaluation to function as a scientific instrument. Non-deterministic initialization or rule selection may lead to inconsistent verdicts for the same mechanism, undermining falsifiability.*

Remark 13. *In Chamber XXIX, all stochastic choices (including initial configuration selection and rule application order) are governed by a seeded pseudo-random generator. The seed may be fixed globally or derived deterministically from the mechanism identifier, ensuring reproducible refinement trajectories.*

Definition 16 (Meta-Instability)**.** *A mechanism exhibits meta-instability if its refinement sequence temporarily enters a recurrent or near-recurrent configuration that is subsequently destroyed by continued refinement.*

Remark 14. *Meta-instability is a transient phenomenon. It does not constitute structural closure, as recurrence fails to persist under refinement.*

Remark 15. *Meta-instability is recorded in Chamber XXIX as diagnostic metadata only. Its detection does not alter the terminal verdict assigned to a mechanism.*

Definition 17 (Terminal Verdicts)**.** *The verdict space of Chamber XXIX is strictly binary:*

$$\texttt{PROTO-CLOSED} \quad \textit{or} \quad \texttt{STRUCTURAL-COLLAPSE}.$$

No intermediate or hybrid verdict classes are permitted.

Remark 16. *A mechanism is assigned* `PROTO-CLOSED` *if and only if it exhibits structural closure as defined in Definition 7 of the main text. All other outcomes, including those involving transient recurrence, result in* `STRUCTURAL-COLLAPSE`*.*

Remark 17. *This discipline preserves the monotonicity of collapse: once a mechanism fails to sustain recurrence under refinement, no subsequent diagnostic observation may reverse that verdict.*

Proposition 8. *Meta-instability does not survive structural refinement and therefore cannot constitute a preserved object under collapse.*

Proof. By definition, meta-instability corresponds to recurrence that fails to persist under continued refinement. Structural closure requires recurrence to be stable under refinement. Therefore, meta-instability is incompatible with preservation under collapse. □

Remark 18. *The detection of meta-instability may be useful for exploratory analysis of mechanism behavior, but it carries no ontological or foundational status. Only proto-closure corresponds to structural persistence.*

Remark 19. *This separation between diagnostic phenomena and terminal verdicts prevents category errors in which transient numerical or symbolic regularities are mistaken for fundamental structure.*

X.6 Relation to Chamber XII Collapse Modes

Chamber XXIX provides a structural refinement of the collapse phenomena empirically observed in Chamber XII. In Chamber XII, collapse was detected through numerical signatures indicating loss of stability, including the absence of invariant outcomes under refinement and the convergence of distinct numerical candidates to indistinguishable collapse profiles.

Chamber XXIX demonstrates that these numerical collapse signatures correspond to the absence of proto-closure at the mechanism level. Mechanisms that fail to exhibit structural closure under refinement are classified as `STRUCTURAL-COLLAPSE`, and their numerical realizations necessarily manifest the collapse modes recorded in Chamber XII.

Conversely, Chamber XXIX establishes that numerical stability alone is insufficient for preservation: mechanisms may exhibit transient numerical or symbolic regularity while still undergoing structural collapse. This explains the Chamber XII observation that both canonical constants and grammar-generated candidates collapse indistinguishably, as all such objects correspond to mechanisms lacking proto-closure.

Taken together, the two chambers show that collapse in the UNNS substrate is determined at the structural level and merely reflected in numerical behavior. Chamber XII provides empirical detection of collapse, while Chamber XXIX identifies its generative cause.